Properties of σ and κ Production Amplitudes

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Our method of analysis, which led to existence of σ and κ mesons, is reviewed and examined from a viewpoint of general S-matrix. It is shown that the method is consistent with the constraints from chiral symmetry and unitarity. Accordingly the long-believed common analyses of $\pi\pi(K\pi)$ scattering and production processes, based on elastic unitarity, prove to lose its theoretical base. The observed phase motion by 180 degrees of σ shows also the validity of our method.

§1. Introduction

The existence of the iso-singlet scalar σ meson had been a longstanding problem of hadron physics. Conventional analyses[1] of $\pi\pi$ scattering phase shift[2] lead to a conclusion of no σ existence, while recent reanalyses[3] including ours suggest the existence of light σ(450-600). The result of the previous analysis with no σ existence was pointed out[4] to be not correct, since in this analysis[1] there is no consideration on the cancellation mechanism between σ amplitude and non-resonant $\pi\pi$ amplitude, which is guaranteed by chiral symmetry.

It is remarkable that, in contrast with the spectra of $\pi\pi$ scattering, the clear peak structure has been observed in mass region of $m_{\pi\pi} \sim 500$ MeV in the various $\pi\pi$ production processes, such as $J/\psi \to \omega\pi\pi$,[5)-8)] $p\bar{p} \to 3\pi^0$,[9),10)] $D^+ \to \pi^+\pi^-\pi^-$[11)] and $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$,[12)] and this structure is shown to be well reproduced by the Breit-Wigner amplitude of σ meson. Presently firm evidences[13)] of σ seem to be accumulated, and the column of σ in particle data group table is corrected as "$f_0(600)$ or σ" in the newest'02[14)] edition in place of f_0(400–1200) or σ in the '96–'00 editions.

There are now hot controversies on the existence of $I = 1/2$ scalar κ meson, to be assigned as a member of σ nonet. Reanalyses[15),16)] of $K\pi$ scattering phase shift[17)] suggest existence of the $\kappa(900)$, while no κ is insisted in ref. 18). The existence of κ is again suggested strongly in $K\pi$ production process of $D^+ \to K^-\pi^+\pi^+$[19)] and $J/\psi \to \bar{K}^{*0}K^+\pi^-$[7),8),20)], similarly to the case of σ.

In the analyses of the $\pi\pi$ (or $K\pi$) production processes mentioned above, the amplitudes are parametrized by a coherent sum of the Breit-Wigner amplitudes including σ (or κ) and of the non-resonant $\pi\pi$ (or $K\pi$) production amplitude. This parametrization method is called VMW method.[21),22)] However, this method was strongly criticized[23),24)] from the conventional viewpoint, so called "universality argument."[1)] In this argument it is insisted that all the $\pi\pi(K\pi)$ production amplitude $\mathcal{F}$ and the scattering amplitude $\mathcal{T}$ have a common phase due to the Watson final state interaction theorem (being based upon the elastic unitarity), and both the

$\pi\pi(K\pi)$ production and scattering processes must be analyzed together with the common phase of the amplitudes.

In this talk we shall review our method of analysis and examine the the relation between $\mathcal{T}$ and $\mathcal{F}$ from a viewpoint of generalized S matrix. As a result, we emphasize that the $\mathcal{F}$ is, in principle, independent from the $\mathcal{T}$ and that the analyses of production processes should be done independently of scattering processes. Accordingly our method of analyses, VMW method, along this line of thought is consistent [25] with all the constraints from unitarity and chiral symmetry. The experimental phase motion of the production amplitude is also examined, and the above criticisms on VMW method will be shown not to be correct also experimentally.

§2. Properties of $\pi\pi/K\pi$ Scattering amplitude

We first review our reanalysis [3] of δ_S^0, $\pi\pi$ scattering phase shift of $I = 0$ S wave amplitude, obtained by CERN-Munich. [2] The applied method is Interfering Amplitude method, where the total δ_S^0 below $m_{\pi\pi} \simeq 1$GeV is represented by the sum of the component phase shifts,

$$\delta_S^0 = \delta_\sigma + \delta_{BG} + \delta_{f_0}. \tag{2·1}$$

The δ_σ and δ_{f_0} are, respectively, contributions from σ and $f_0(980)$ Breit-Wigner amplitudes. The δ_{BG} is from non-resonant repulsive $\pi\pi$ amplitude, which is taken phenomenologically of hard-core type, $\delta_{BG} = -p_1 r_c$ ($p_1 = \sqrt{s/4 - m_\pi^2}$ being the CM momentum of π).

The experimental δ_S^0 passes through 90° at $\sqrt{s}(= m_{\pi\pi}) \sim 900$MeV. This is explained by the cancellation between attractive δ_σ and repulsive δ_{BG}. (See ref. 3).) The mass and width of σ is obtained as $m_\sigma = 585 \pm 20$MeV and $\Gamma_\sigma = 385 \pm 70$MeV.

Note that the above cancellation is shown [4] to come from chiral symmetry in the linear σ model (LσM): The $\pi\pi$ scattering $A(s,t,u)$ amplitude in LσM is given by

$$A(s,t,u) = \frac{(-2g_{\sigma\pi\pi})^2}{m_\sigma^2 - s} - 2\lambda = \frac{s - m_\pi^2}{f_\pi^2} + \frac{1}{f_\pi^2}\frac{(s - m_\pi^2)^2}{m_\sigma^2 - s}, \tag{2·2}$$

as a sum of the σ amplitude A_σ, which is strongly attractive, and of the non-resonant $\pi\pi$ amplitude $A_{\pi\pi}$ due to the $\lambda\phi^4$ interaction, which is stongly repulsive. They cancel with each other following the relation of LσM, $g_{\sigma\pi\pi} = f_\pi\lambda = (m_\sigma^2 - m_\pi^2)/(2f_\pi)$, and the small $\mathcal{O}(p^2)$ Tomozawa-Weinberg (TW) amplitude and its correction are left. The $A_\sigma(A_{\pi\pi})$ corresponds to $\delta_\sigma(\delta_{BG})$. Actually, the theoretical predictions for δ_{NR}^0 and δ_{NR}^2, obtained by unitarizing $A_{\pi\pi}$, $A(t,s,u)$ and $A(u,t,s)$ in LσM, are consistent with our δ_{BG} of hard core type in our phase shift analysis, [3] and with experimental δ_0^2, [26] respectively (See ref. 4), 27)).

Thus, it is shown that the σ Breit-Wigner amplitude with non-derivative ($\mathcal{O}(p^0)$) $\pi\pi$-coupling requires at the same time the strong ($\mathcal{O}(p^0)$) repulsive $\pi\pi$ interaction to obtain the small $\mathcal{O}(p^2)$ TW amplitude, satisfying chiral symmetry. This is the origin of δ_{BG} in our phase shift analysis.[*]

[*] There was an argument [23] that a broad resonance with mass 1GeV, denoted as $f_0(1000)$ [2]

The similar cancellation is also expected to occur in $K\pi$ scattering since K has also a property of Nambu-Goldstone boson. The experimental $I = 1/2$ S wave phase shift $\delta_S^{1/2}$ passes through 70 degrees at about $m_{K\pi} \sim 1.3$GeV. This $\delta_S^{1/2}$ is parametrized by introducing the κ Breit-Wigner phase shift δ_κ and its compensating repulsive non-resonant $K\pi$ phase shift $\delta_{BG}^{Non.Res}$ as $\delta_S^{1/2} = \delta_\kappa + \delta_{BG}^{Non.Res} + \delta_{K_0^*(1430)}$. The fit [16] to $\delta_S^{1/2}$ below 1.6 GeV by LASS [17] gives the mass and width of κ meson [16] as $m_\kappa = 905\ {}^{+65}_{-30}$ MeV and $\Gamma_\kappa = 545\ {}^{+235}_{-110}$ MeV.

As we explained above, because of the chiral cancellation mechanism, the $\pi\pi/K\pi$ scattering amplitude $\mathcal{T}$ has the spectra strongly suppressed in the threshold region. Correspondingly $\mathcal{T}$ shows slowly varying phase motion in the σ/κ mass region(, totally about 90 degrees being much smaller than 180 degrees given by the $\sigma(\kappa)$ Breit-Wigner amplitude itself). We consider whether these two features of $\mathcal{T}$ are also valid in general $\pi\pi/K\pi$ production amplitudes $\mathcal{F}$ or not, in the following sections.

§3. Method of Analyses of $\pi\pi/K\pi$ Production Processes

(*Essence of VMW method*) The analyses of $\pi\pi/K\pi$ production processes quoted in §1 are done following the VMW method. [21), 22)] Here we explain our basic physical picture on strong interactions and the essential point of this method.

The strong interaction is a residual interaction of QCD among all color-neutral bound states of quarks(q), anti-quarks($\bar{q}$) and gluons(g). These states are denoted as ϕ_i, and the strong interaction Hamiltonian $\mathcal{H}_{\rm str}$ is described by these ϕ_i fields. It should be noted that, from the quark physical picture, [22] unstable particles as well as stable particles, if they are color-singlet bound states, should be equally treated as ϕ_i-fields on the same footing.

$$\begin{aligned} \mathcal{H}_{\rm str} &= \mathcal{H}_{\rm str}(\phi_i) \\ \{\ \phi_i\ \} &= \{\text{color singlet bound states of } q, \bar{q} \text{ and } g\}. \end{aligned} \tag{3·1}$$

The time-evolution by $\mathcal{H}_{\rm str}(\phi_i)$ describes the generalized S-matrix. Here, it is to be noted that, if $\mathcal{H}_{\rm str}$ is hermitian, the unitarity of S matrix is guaranteed.

The bases of generalized S-matrix are the configuration space of these multi-ϕ_i states.

$$\begin{array}{c} S \text{ matrix bases} \\ \{\text{multi}-\phi_i-\text{states}\} \end{array} = \left\{ \begin{array}{l} |\omega\pi\pi\rangle, \underline{|\omega\sigma\rangle, |\omega f_2\rangle, |b_1\pi\rangle, \cdots, |J/\psi\rangle}, \\ |N\pi\rangle, |N\pi\pi\rangle, \underline{|\Delta\rangle, |\Delta\pi\rangle, |\Delta\sigma\rangle, \cdots}, \\ \cdots\cdots \end{array} \right\}, \tag{3·2}$$

where the states relevant for $J/\psi \to \omega\pi\pi$ decay and $N\pi$ scattering are respectively shown in 1st and 2nd lines as examples. The states including unstable particles shown with underlines are equally treated with non-resonant states $|\omega\pi\pi\rangle$ and $|N\pi\rangle, |N\pi\pi\rangle$.

The relevant $J/\psi \to \omega\pi\pi$ decay process is described by a coherent sum of different non-diagonal elements of various 2-body decay amplitudes, ${}_{out}\langle\omega\sigma|J/\psi\rangle_{in}$,

or $\epsilon(900)$, [1] instead of light σ, exists. However, in this argument the above cancellation mechanism is not considered, and their conclusion of no existence of light σ is not correct.

$_{out}\langle\omega f_2|J/\psi\rangle_{in}$, $_{out}\langle b_1\pi|J/\psi\rangle_{in}$, $\cdots$, and a non-resonant 3-body($\omega\pi\pi$) decay amplitude$_{out}\langle\omega\pi\pi|J/\psi\rangle_{in}$. They have mutually independent coupling strengths, $r_{\psi\sigma}$, $r_{\psi f_2}$, $r_{\psi b_1}$, $\cdots$, $r_{\psi\pi\pi}$.*) The $\mathcal{H}_{\rm str}$ induces the various final state interaction, reducing to the strong phases of the corresponding amplitudes, $\theta_{\psi\sigma}$, $\theta_{\psi f_2}$, $\cdots$. (See Fig.1.) These phases are related with the phases of diagonal elements θ_ψ, θ_σ, $\cdots$ which are unknown. Thus, we may treat phenomenologically the formers as independent parameters from the latters.

The remaining problem is how to treat unstable particles, as there is no established field-theoretical method for this problem. In VMW method the decay of the unstable particles are treated intuitively by the replacement of propagator,

$$\begin{array}{ccc} \text{Stable particle} & & \text{Unstable particle} \\ \dfrac{1}{m_\sigma^2 - s - i\epsilon} & \xrightarrow{\text{Strong Int.}} & \Delta_\sigma(s) = \dfrac{m_\sigma\Gamma_\sigma}{m_\sigma^2 - s - im_\sigma\Gamma_\sigma(s)}, \end{array} \qquad (3\cdot3)$$

where we take the case of σ as an example. Here we should note that the imaginary part of the denominator $im_\sigma\Gamma_\sigma(s)$ does not come from the $\pi\pi$ final state interaction (or the repetition of the virtual $\pi\pi$ loops), but from the decay of σ to $\pi\pi$ state.**)

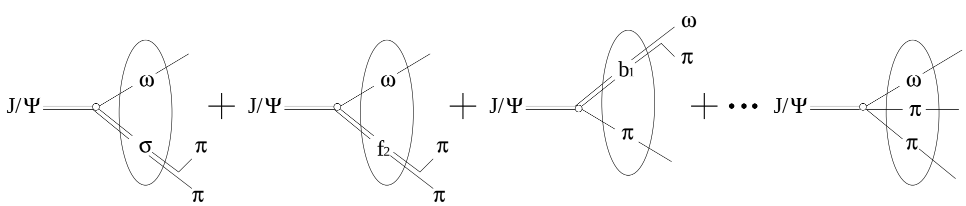

Fig. 1. The final state interactions in $J/\psi\to\omega\pi\pi$. The amplitude is a superposition of different S matrix elements, such as $J/\psi\to\omega\sigma$, $J/\psi\to\omega f_2$, $J/\psi\to b_1\pi$, $\cdots$, $J/\psi\to\omega(\pi\pi)_{Non.Res.}$. The ellipses represent the final state interactions, and the corresponding amplitudes have independent strong phases.

As a result, the effective $\omega\pi\pi$ amplitude is given by

$$\begin{aligned}
\mathcal{F}_{\omega\pi\pi} &= \mathcal{F}_{\omega\sigma} + \mathcal{F}_{\omega f_2} + \mathcal{F}_{b_1\pi} + \cdots + \mathcal{F}_{\omega(\pi\pi)_{Non.Res.}}, \qquad (3\cdot4)\\
\mathcal{F}_{\omega\sigma} &= {}_{out}\langle\omega\sigma|J/\psi\rangle_{in}\Delta_\sigma(s) = r_{\psi\sigma}e^{i\theta_{\psi\sigma}}\frac{m_\sigma\Gamma_\sigma}{m_\sigma^2 - s - im_\sigma\Gamma_\sigma(s)}\\
\mathcal{F}_{\omega f_2} &= {}_{out}\langle\omega f_2|\ J/\psi\ \rangle_{in}\Delta_{f_2}(s) = r_{\psi f_2}e^{i\theta_{\psi f_2}}\frac{m_{f_2}\Gamma_{f_2}N_{\pi\pi}(s,\cos\theta)}{m_{f_2}^2 - s - im_{f_2}\Gamma_{f_2}(s)}\\
\mathcal{F}_{\omega b_1} &= {}_{out}\langle\omega b_1|J/\psi\rangle_{in}\Delta_{b_1}(s) = r_{\psi b_1}e^{i\theta_{\psi b_1}}\frac{m_{b_1}\Gamma_{b_1}}{m_{b_1}^2 - s - im_{b_1}\Gamma_{b_1}(s)}\\
&\cdots
\end{aligned}$$

*) The strengths and phases of respective amplitudes are considered to be determined by quark dynamics. However, we treat them independent in phenomenological analyses.

**) In non-relativistic quantum mechanics, the decay of the unstable particle can be described by the WF with imaginary part in the energy, $e^{-iE_0t}\to e^{-i(E_0-i\Gamma/2)t}$. Correspondingly, the propagator is replaced as $\frac{1}{E_0-E}\to\frac{1}{E_0-E-i\Gamma/2}$. Similarly, in the Feynman propagator, $\Delta_F(x) = \int\frac{d^4k}{(2\pi)^4}\frac{e^{ikx}}{m^2+k^2-i\epsilon} = \int\frac{d^3\mathbf{k}}{i(2\pi)^3 2\omega}e^{-i\omega|x_0|}e^{i\mathbf{k}\cdot\mathbf{x}}$, when we replace $e^{-i\omega|x_0|}$ by $e^{-i(\omega-i\Gamma/2)|x_0|}$, we obtain the propagator $\frac{1-i\Gamma/(2\omega)}{m^2+k^2-\Gamma^2/4-i\omega\Gamma}$, which may be approximated as $\frac{1}{m^2-s-im\Gamma(s)}$ in Eq. (3·3).

$$\mathcal{F}_{\omega(\pi\pi)}^{Non.Res.} = {}_{out}\langle\omega(2\pi)_{N.R.}|J/\psi\rangle_{in} \quad = \quad r_{2\pi}^{N.R.} e^{i\theta_{2\pi}^{N.R.}} \quad , \tag{3·5}$$

where $\mathcal{F}_{\omega\sigma}$, $\mathcal{F}_{\omega f_2}$, $\cdots$ corresponds to the generalized S matrix element ${}_{out}\langle\omega\sigma|\ J/\psi\ \rangle_{in}$, ${}_{out}\langle\omega f_2|\ J/\psi\ \rangle_{in}$, respectively. (The extra Breit-Wigner factor comes from the prescription Eq. (3·3), and $N_{\pi\pi}$ is angular function of $f_2 \to \pi\pi$ D-wave decay.) They have mutually-independent couplings, $r_{\psi\sigma}$, $r_{\psi f_2}$, $r_{\psi b_1}$, $\cdots$ and $r_{\psi 2\pi}^{N.R.}$. The strong phases $\theta_{\psi\sigma}$, θ_{f_2}, etc. are treated as free parameters, as we explained previously. The amplitude given by Eqs. (3·4) and (3·5) is consistent with the S-matrix unitarity(generalized unitarity). This parametrization method is called as VMW method.

(*Chiral constraint and threshold suppression*) In $\pi\pi$ scattering the derivative-coupling property of Nambu-Goldstone π-meson requires the suppression of the amplitude $\mathcal{T}_{\pi\pi}$ near threshold,

$$\mathcal{T}_{\pi\pi} \sim -p_{\pi 1}\cdot p_{\pi 2} \to m_\pi^2 \sim 0 \quad \text{at} \quad s \to 4m_\pi^2. \tag{3·6}$$

This chiral constraint requires, as was explained in §2, the strong cancellation between the σ amplitude $\mathcal{T}_\sigma$ and the non-resonant $\pi\pi$ amplitude $\mathcal{T}_{2\pi}$, which means the strong constraints, $r_{2\pi} \simeq -r_\sigma$, $\theta_\sigma \simeq \theta_{2\pi}$, in the corresponding formulas to Eqs. (3·4) and (3·5). The amplitude has zero close to the threshold and no direct σ Breit-Wigner peak is observed in $\pi\pi$ mass spectra.

On the other hand, for general $\pi\pi$ production processes the parameters r_i and θ_i are independent of those in $\pi\pi$ scattering, since they are concerned with different S-matrix elements. Especially we can expect in the case of "σ-dominance", $r_\sigma \gg r_{2\pi}$, the $\pi\pi$ spectra show steep increase from the $\pi\pi$ threshold, and the σ Breit-Wigner peak is directly observed. This situation seems to be realized in $J/\psi \to \omega\pi\pi$ and $D^+ \to \pi^-\pi^+\pi^+$.

Here we should note that the chiral constraint on r_σ and $r_{2\pi}$ does not work generally in the production processes with large energy release to the $\pi\pi$ system. We explain this fact in case of Υ decays.[25] Here we take $J/\psi \to \omega\pi\pi$ as an example. We consider a non-resonant $\pi\pi$ amplitude of derivative-type $\mathcal{F}_{\rm der}$,

$$\mathcal{F}_{\rm der} \sim P_\psi\cdot p_{\pi 1} P_\psi \cdot p_{\pi 2}/M_\psi^2, \tag{3·7}$$

where $P_\psi(p_{\pi i})$ is the momentum of J/ψ (emitted pions).[*)] This type of amplitude satisfies the Adler self-consistency condition, $\mathcal{F} \to 0$ when $p_{\pi 1\mu} \to 0$, and consistent with the general chiral constraint. However, this zero does not appear in low energy region of actual s-plane. At $\pi\pi$ threshold (where $s = 4m_\pi^2$), $p_{\pi 1\mu} = p_{\pi 2\mu}$ and $P_\psi \cdot p_{\pi i}/M_\psi = E_{\pi i} = (M_\psi^2 - m_\omega^2 + 4m_\pi^2)/(4M_\psi)$ ($E_{\pi i}$ being the energy of emitted pion), and thus,

$$\mathcal{F}_{\rm der} \to \{\ (M_\psi - (m_\omega^2 - 4m_\pi^2)/M_\psi)/4\ \}^2 \gg m_\pi^2 \quad \text{at} \quad s \to 4m_\pi^2. \tag{3·8}$$

The amplitude (3·7) is not suppressed near $\pi\pi$ threshold, and correspondingly there is no strong constraint between $r_{\psi\sigma}e^{i\theta_{\psi\sigma}}$ and $r_{2\pi}e^{i\theta_{2\pi}}$ leading to the threshold suppression. This is quite in contrast with the situation in $\pi\pi$ scattering, Eq. (3·6).

[*)] The equation (3·7) is obtained by the chiral symmetric effective Lagrangian, $\mathcal{L}_d = \xi_d \partial_\mu\partial_\nu\psi_\lambda\omega_\lambda(\partial_\mu\pi\partial_\nu\pi + \partial_\mu\sigma\partial_\nu\sigma)$. The possible origin of this effective Lagrangian is discussed in ref.[25]. There occurs no one σ-production amplitude, cancelling the 2π amplitude, in this Lagrangian.

(*"Universality" of $\mathcal{T}_{\pi\pi}$: threshold behavior*) Conventionally all the production amplitudes $\mathcal{F}_{\pi\pi}$, including the $\pi\pi$ system in the final channel, are believed [1] to take the form proportional to $\mathcal{T}_{\pi\pi}$ as

$$\mathcal{F}_{\pi\pi} = \alpha(s)\mathcal{T}_{\pi\pi}; \quad \alpha(s) : \text{slowly varying real function}, \qquad (3\cdot9)$$

where $\alpha(s)$ is supposed to be a slowly varying real function. This implies that $\mathcal{F}$ and $\mathcal{T}$ have the same phases and the same structures (the common positions of poles, if they exist). The equation (3·9) is actually applied to the analyses of various production processes [1, 24, 28], and it was the reason of overlooking σ for almost 20 years in the 1976 through 1994 editions of Particle Data Group tables.

The equation (3·9) is based on the belief that low energy $\pi\pi$ chiral dynamics is also applicable to $\pi\pi$ production processes with small s, leading to the threshold suppression of spectra, as in Eq. (3·6), in all production processes because of the Adler zero in $\mathcal{T}_{\pi\pi}$. This is apparently inconsistent with experimental data.

So, in order to remove the Adler zero at $s = s_0$ and to fit the experimental spectra, one is forced to modify [1, 24] the form of $\alpha(s)$ by multiplying artificially the rapidly varying factor $1/(s - s_0)$ without any theoretical reason.

Here I should like to stress the physical meaning of an example, Eq. (3·7), that Adler zero condition, even in the isolated final 2π system, does not necessarily lead to the threshold suppression. This implies that the above mentioned ad hoc prescription [24] to get rid of the undesirable zero near threshold of $\mathcal{F}$ becomes not necessary, if we take the new dynamics, which would appear with large energy release, duely into account.

§4. Phases of Production Amplitudes

(*Generalized S matrix and phase of production amplitude*) It is often discussed that in order to confirm the existence of a resonant particle, it is necessary to observe the corresponding phase motion $\Delta\delta \sim 180°$ of the amplitude. In the $\pi\pi$ P wave amplitude a clear phase motion $\Delta\delta \sim 180°$ due to ρ meson Breit-Wigner amplitude is observed. However, in the case of σ meson, because of the chiral cancellation mechanism (explained in §2), and of its large width, its phase motion cannot be observed directly in the $\pi\pi$ scattering amplitude. Because, *in production processes, the amplitudes are a sum of various S matrix elements (as explained in §3), the pure σ Breit-Wigner phase motion may be generally difficult to be observed.* However, in some exceptional cases, when the amplitude is dominated by σ, it may be directly observed.

On the other hand, it is widely believed that all the $\pi\pi$ production amplitudes $\mathcal{F}$ have the same phase as that of the scattering amplitude. In the relevant $J/\psi \to \omega\pi\pi$ decay, it is often argued that the $\mathcal{F}$ near $\pi\pi$ threshold must take the same phase as the scattering phase δ, since in this energy region the $m_{\omega\pi}(\sim M_\psi)$ is large and $\pi\pi$ decouples from ω in the final channel. [31]

However, *this belief (or Eq. (3·9)) comes from an improper application of the elastic unitarity condition, which is not applicable to the production processes*, where the freedom of various strong phases, $\theta_{\psi\sigma}$, $\theta_{\psi b_1}$, $\theta_{\psi f_2}$, $\cdots$ is allowed from the *gener-*

alized unitarity condition.

Because of the above strong phases, generally $\mathcal{F}$ have different phases from $\mathcal{T}$. The $\mathcal{F}$ has the same phase as $\mathcal{T}$ only when the final $\pi\pi$ systems are isolated in strong interaction level. For 3-body decays such as $J/\psi \to \omega\pi\pi$ and $D^- \to \pi^-\pi^+\pi^+$, the above condition is not satisfied. Actually, the large strong phases in J/ψ and D decays are suggested experimentally. In order to reproduce the branching ratios of $J/\psi \to 1^-0^-$ decays (that is, $J/\psi \to \omega\pi^0, \rho\pi, K^*\bar{K}, \cdots,$) it is necessary to introduce a large relative strong phase [29] $\delta_\gamma = arg\frac{a_\gamma}{a} = 80.3^\circ$ between the effective coupling constants of three gluon decay a and of one photon decay a_γ. A similar result is also obtained in $J/\psi \to 0^-0^-$ decays. A large relative phase between $I = 3/2$ and $I = 1/2$ amplitudes of $D \to K\pi$ decays is observed: $\delta_{3/2}(m_D) - \delta_{1/2}(m_D) = (96 \pm 13)^\circ$, [30] (while in $B \to D\pi, D\rho, D^*\pi$ decays rather small relative phases are obtained). By considering these results we expect existence of not small strong phases $\theta_{\psi\sigma}$, $\theta_{\psi b_1}$, $\cdots$ coming from $\sigma\omega, b_1\pi, \cdots$ rescatterings in J/ψ decays.

According to these works [29), 30)] M_ψ (M_D) are not sufficiently large for making $\pi\pi$ decouple from ω ($\pi_1^-\pi_2^+$ from π_3^+). The $\pi\pi$ elastic unitarity is not valid in the amplitude $\mathcal{F}$ Eq. (3·4), which takes the different phase from $\mathcal{T}$. Similar consideration is also applicable to the $J/\psi \to K^*K\pi$ and $D^- \to K^-\pi^+\pi^+$, and the corresponding $K\pi$ production amplitudes $\mathcal{F}$ take different phases from the phase of $K\pi$ scattering amplitude $\mathcal{T}$.

(180° *phase motion of σ-meson observed in $D^+ \to \pi^-\pi^+\pi^+$*) Recently a method extracting the σ phase motion from Dalitz plot data of $D^+ \to \pi_1^-\pi_2^+\pi_3^+$ decays is presented in ref. [32)], where the interference between the $f_2(1275)$ Breit-Wigner amplitude in $s_{12} = m^2_{\pi_1^-\pi_2^+} \simeq (1.275\text{GeV})^2$ region and the remaining S-wave component in $s_{13} = m_{\pi_1^-\pi_3^+} \simeq (0.5\text{GeV})^2$ region is used. The actual analysis of the E791 data gives the clear 180 degrees phase motion in $s_{13} \simeq 0.25\text{GeV}^2$ region which is consistent with that [33)] of the σ Breit-Wigner amplitude. This phase motion is completely different from the $\pi\pi$ scattering phase shift, where the phase moves only 90 degrees below 900 MeV. Thus, *it is experimentally confirmed that the phase of $\pi\pi$ production amplitude $\mathcal{F}$ in this process is different from that in $\pi\pi$ scattering amplitude $\mathcal{T}$* and that *the $\pi\pi$ elastic unitarity does not work in D decays.* The 180 degrees phase motion seems to suggest experimentally the σ is not the state from final $\pi\pi$ interaction but corresponds to the real entity in quark level forming the general S-matrix bases.

Similar result is also obtained for κ in $D^+ \to K^-\pi^+\pi^+$. The large phase motion of $K\pi$ S wave component around κ energy region is suggested, which is much larger than the $K\pi$ scattering phase shift obtained by LASS experiment. [17)]

§5. Concluding Remarks

For many years it had been believed that both the scattering and the production $\pi\pi/K\pi$ amplitudes in the low mass region with $\sqrt{s} < 1\text{GeV}$ have the same structures (universality of scattering amplitudes), and that analyses of any production process should be done together with the scattering process. Because of this belief the

peak structures (due to σ/κ meson) in the various production processes had been regarded as mere backgrounds. However, in this talk we have explained that the above belief is not correct and that the production processes should generally be treated independently from the scattering process. The essential reason of this(, as is explained in §3,) is that as the basic fields of expanding the S matrix, the "bare" fields of σ and κ (as the bound states of quarks), as well as the π and K fields, should be taken into account.

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